

Mixed Linear Models, Module 3: Drying of beech wood - a case study, part I.

Per Bruun Brockhoff

pmb@kvl.dk

The data

- 20 planks was dried in a certain period of time.
- The humidity percentage was measured at:
 - 5 depths
 - * depth 1: close to the top
 - * depth 5: in the center
 - * depth 9: close to the bottom
 - * depth 3: between 1 and 5
 - * depth 7: between 5 and 9
 - 3 widths
 - * width 1: close to the side
 - * width 3: in the center
 - * width 2: between 1 and 3
- Aim: To investigate the effect of drying of beech wood on the humidity percentage.

Content of Module 3:

- Data presentation
- The basic factor structure
- Explorative analysis
- Test of effects/model reduction
- Post hoc analysis and results summary
- Statistical analysis: the overall approach

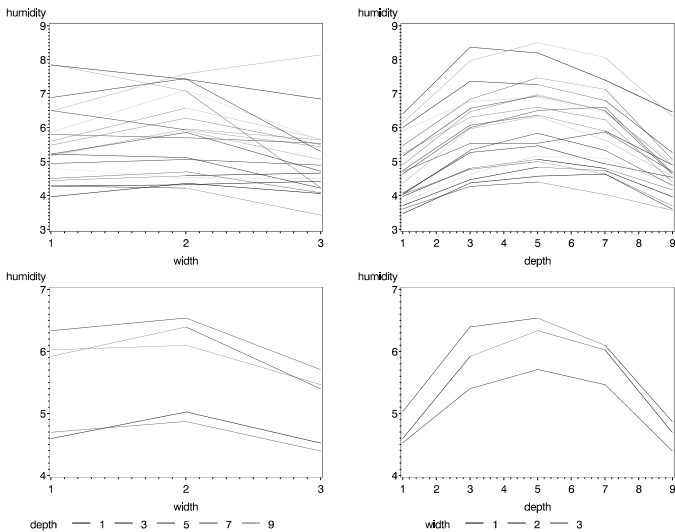
The data, cont.

Planks	Width 1 Depth					Width 2 Depth					Width 3 Depth				
	1	3	5	7	9	1	3	5	7	9	1	3	5	7	9
1	3.4	4.9	5.0	4.9	4.0	4.1	4.7	5.2	4.6	4.3	4.4	4.8	5.0	4.9	4.2
2	4.3	5.5	6.2	5.4	4.7	3.9	5.6	5.7	5.5	4.9	4.0	4.7	4.5	3.9	4.0
3	4.2	5.5	5.6	6.3	4.5	5.4	6.2	6.1	6.4	5.2	4.5	4.9	4.9	4.9	4.4
4	4.4	6.0	7.1	6.9	4.6	4.6	6.1	6.6	6.5	4.7	4.9	5.9	5.8	6.4	4.7
5	3.9	4.7	5.2	5.0	3.7	4.2	5.2	5.4	4.8	3.9	4.0	4.4	4.4	4.1	3.5
6	4.6	5.9	6.3	5.8	4.8	5.9	7.3	6.9	6.9	4.4	5.2	5.7	6.6	6.0	4.0
7	3.9	5.6	6.0	5.3	5.0	4.9	6.9	7.1	6.1	4.5	4.3	5.4	5.9	5.5	4.2
8	3.9	4.5	5.3	5.6	4.7	3.7	4.9	4.8	4.9	4.3	3.8	4.5	5.4	4.8	4.0
9	3.6	4.1	4.0	4.4	3.7	3.8	5.1	5.0	4.6	3.3	3.0	3.9	4.7	4.9	3.8
10	6.5	8.7	9.5	7.9	6.6	6.9	8.9	7.4	7.0	6.9	5.8	7.5	7.7	7.3	5.9
11	3.7	5.2	5.5	5.9	4.4	4.7	5.8	5.7	4.9	4.2	3.7	5.0	6.3	5.2	4.3
12	4.3	5.8	6.2	5.2	4.4	4.8	6.7	7.0	6.1	5.2	5.1	5.7	5.9	6.4	5.1
13	6.5	8.8	9.1	8.9	6.0	5.9	7.5	8.4	7.9	5.7	4.0	4.2	4.9	4.6	3.5
14	4.4	6.2	6.7	6.4	4.3	5.7	7.0	7.4	7.3	5.5	4.6	6.2	6.8	5.8	4.9
15	5.5	7.1	7.5	6.9	5.4	6.4	8.4	8.9	8.1	6.1	6.5	8.4	9.1	9.2	7.5
16	5.2	6.0	6.2	6.6	5.3	6.6	7.6	7.8	7.7	5.8	5.9	6.7	6.7	5.0	3.9
17	3.7	4.5	5.0	4.5	3.7	3.7	4.4	4.8	4.4	4.3	3.7	4.5	4.7	5.3	3.9
18	6.0	7.4	7.8	7.5	5.7	6.9	8.6	8.8	7.5	5.4	5.1	6.1	5.2	5.4	4.7
19	3.8	4.6	4.8	4.4	3.8	3.7	4.7	4.7	4.3	3.7	3.3	3.5	3.7	3.4	3.2
20	6.1	7.4	7.7	6.7	4.6	4.7	6.3	7.1	6.5	5.1	4.7	6.0	6.0	6.3	4.2

Identify the factors and their structure

- Factors:
 - I: 300 experimental units
 - 0: 1 overall level
 - depth: 5 levels
 - width: 3 levels
 - plank: 20 levels
 - Crossed "treatment" structure: width × depth: 15 levels
- plank is a natural block effect.

Initial explorative analysis



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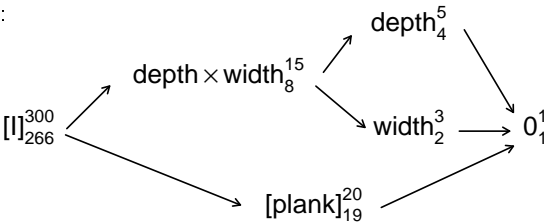
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Identify the factors and their structure, cont.

- Factor structure:



- Model:

$$Y_i = \mu + \alpha(\text{width}_i) + \beta(\text{depth}_i) + \gamma(\text{width}_i, \text{depth}_i) + d(\text{plank}_i) + \epsilon_i,$$

$$d(j) \sim N(0, \sigma_{\text{plank}}^2), \quad \epsilon_i \sim N(0, \sigma^2).$$

$$j = 1, \dots, 20, \quad i = 1, \dots, 300$$

Test of overall effects/model reduction

- Fixed effects ANOVA tables:

Source of variation	Numerator degrees of freedom	Denominator degrees of freedom	Sums of squares	Mean squares
depth	4	266	78.26	<0.0001
width	2	266	29.65	<0.0001
depth*width	8	266	1.08	0.3745

Source of variation	Numerator degrees of freedom	Denominator degrees of freedom	Sums of squares	Mean squares
depth	4	274	78.07	<0.0001
width	2	274	29.57	<0.0001

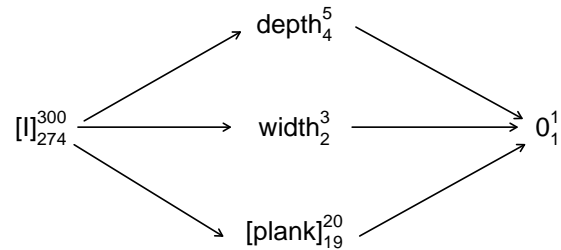
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Final model



- Corresponding to;

$$Y_i = \mu + \alpha(\text{width}_i) + \beta(\text{depth}_i) + d(\text{plank}_i) + \epsilon_i$$

$$d(j) \sim N(0, \sigma_{\text{plank}}^2), \quad \epsilon_{ijk} \sim N(0, \sigma^2).$$

Post hoc analysis and summarizing the results

- Estimates of the fixed parameters:

	Parameter	Estimate	SE	Lower	Upper
Depth 1	$\mu + \beta(1)$	4.7150	0.2361	4.2270	5.2030
Depth 3	$\mu + \beta(2)$	5.9050	0.2361	5.4170	6.3930
Depth 5	$\mu + \beta(3)$	6.1950	0.2361	5.7070	6.6830
Depth 7	$\mu + \beta(4)$	5.8633	0.2361	5.3753	6.3514
Depth 9	$\mu + \beta(5)$	4.6533	0.2361	4.1653	5.1414

	Parameter	Estimate	SE	Lower	Upper
Width 1	$\mu + \alpha(1)$	5.5140	0.2303	5.0352	5.9928
Width 2	$\mu + \alpha(2)$	5.7860	0.2303	5.3072	6.2648
Width 3	$\mu + \alpha(3)$	5.0990	0.2303	4.6202	5.5778

Post hoc analysis and summarizing the results

- Estimates of the variance parameters:

$$\hat{\sigma}_{\text{planks}}^2 = 0.9797, \quad \hat{\sigma}^2 = 0.4047$$

Comparisons of the fixed parameters

- t-test:

$$t = \frac{\hat{\beta}(1) - \hat{\beta}(2)}{SE(\hat{\beta}(1) - \hat{\beta}(2))}$$

- 95% confidence interval:

$$\hat{\beta}(1) - \hat{\beta}(2) \pm t_{.975, 274} SE(\hat{\beta}(1) - \hat{\beta}(2))$$

- Warning: ONLY make comparisons decided for in advance (before seeing the data) this way.
- For multiple comparisons between all levels of a factor: Use an "adjustment" method:
 - Bonferroni
 - Tukey-Kramer
 - Or other methods

Comparisons of the fixed parameters, results

Depth difference	Parameter	Estimate	SE	Lower	Upper	P-value
1-3	$\beta(1) - \beta(2)$	-1.1900	0.1162	-1.5090	-0.8710	<0.0001
1-5	$\beta(1) - \beta(3)$	-1.4800	0.1162	-1.7990	-1.1610	<0.0001
1-7	$\beta(1) - \beta(4)$	-1.1483	0.1162	-1.4673	-0.8294	<0.0001
1-9	$\beta(1) - \beta(5)$	0.06167	0.1162	-0.2573	0.3806	0.9841
3-5	$\beta(2) - \beta(3)$	-0.2900	0.1162	-0.6090	0.02896	0.0943
3-7	$\beta(2) - \beta(4)$	0.04167	0.1162	-0.2773	0.3606	0.9964
3-9	$\beta(2) - \beta(5)$	1.2517	0.1162	0.9327	1.5706	<0.0001
5-7	$\beta(3) - \beta(4)$	0.3317	0.1162	0.01271	0.6506	0.0370
5-9	$\beta(3) - \beta(5)$	1.5417	0.1162	1.2227	1.8606	<0.0001
7-9	$\beta(4) - \beta(5)$	1.2100	0.1162	0.8910	1.5290	<0.0001

Width difference	Parameter	Estimate	SE	Lower	Upper	P-value
1-2	$\alpha(1) - \alpha(2)$	-0.2720	0.08997	-0.4840	-0.05998	0.0077
1-3	$\alpha(1) - \alpha(3)$	0.4150	0.08997	0.2030	0.6270	<0.0001
2-3	$\alpha(2) - \alpha(3)$	0.6870	0.08997	0.4750	0.8990	<0.0001

The overall approach:

- Identify factors and their structure leading to starting model.
- Explorative analysis
- Test of effects/model reduction leading to final model ("Top-down approach")
- Sometimes a "Bottom-up" approach (or a combination) may be relevant.
- Post hoc analysis and results summary
 - Estimates of the variance parameters
 - Estimates of the fixed parameters
 - Comparisons of the fixed parameters
- One important thing missing: **Model diagnostics!**

Comparisons of the fixed parameters, in summary

	Estimate
Depth 9	4.6533 _a
Depth 1	4.7150 _a
Depth 7	5.8633 _b
Depth 3	5.9050 _{bc}
Depth 5	6.1950 _c

	Estimate
Width 3	5.0990 _a
Width 1	5.5140 _b
Width 2	5.7860 _c